TECHNICAL NOTE

EFFECTS OF SLACK CORRECTION ON TENSILE STIFFNESS OF GEOSYNTHETICS

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ABSTRACT

Tensile stiffness and strength are important design parameters of geogrids and geotextiles when they are used in soil reinforcement applications. These design parameters are commonly determined using the wide-width (ASTM D 4595) and multi-rib tensile testing method (ASTM D 6637). It is recognized that these design parameters are actually determined from the slack-corrected (shifted) load-displacement curve instead from an original one. The curve shifting is referred to as slack correction as outlined in Section 11.2.1 of ASTM D 6637. The general perception within the geosynthetic community is that slack correction overestimates geosynthetic tensile stiffnesses, especially those at low strains. This technical note is to show that this general perception is true only if the original load-displacement curve is concave upward over the interval containing those low strains; and false if the original load-displacement curve is either linear or concave downward over the interval containing those low strains.

INTRODUCTION

Tensile stiffness and strengths are important design parameters of geogrids and geotextiles used in soil reinforcement applications. These parameters are commonly determined using the wide-width (ASTM D 4595) and multi-rib tensile testing method (ASTM D 6637). In accordance with the interpretation procedures outlined in Section 11.2.1 of ASTM D 6637, an original load-displacement curve, as shown in Figure 1, for a given geosynthetic specimen is to be subjected to slack correction. The slack correction includes the following steps:

- Determine the maximum load, F_{MAX},, from the original load-displacement curve described by a function, $y = f(x)$, where x and y represent original displacements and loads, respectively;

- Calculate slack tension. y_O = 1.25% times F_{MAX} if F_{MAX} is less or equal 4,000 lbs or y_O = 50 lbs if F_{MAX} is greater than 4,000 lbs (ASTM D 6637);

- Determine slack displacement, x_O, which is the displacement corresponding to y_O from the original load-displacement curve as shown in Figure 1;

- Subtract slack tension from each originally recorded load to obtain the corrected loads;

$$y' = y - y_O \tag{1}$$

- Subtract slack displacement from each originally recorded displacement to obtain the corrected displacements;

$$x' = x - x_O \tag{2}$$

- Use corrected displacements and loads, x' and y', to construct the new displacement curve described by a function $y' = f(x')$. $f(x')$ is referred to as slack-corrected load-displacement curve in this technical note; and

- Determine tensile properties from the slack-corrected load-displacement curve.

Recognizing that slack can be either an inherited property of some geosynthetic products, caused by test setup, or both, it is beyond the scope of this technical note to identify whether the slack is a test-related problem or true behavior of some geosynthetic materials. The worst case is that the slack is an inherited property for some geosynthetic materials but it is mathematically removed in accordance with ASTM D 6637 or physically removed by preloading (ASTM 4595). The general perception within the geosynthetic community is that slack correction overestimates tensile stiffnesses, especially those at low strains. The tensile stiffness is defined as stress (force per unit width) divided by the corresponding strain, and is also referred to as secant modulus. This technical note presents a detailed analysis on how slack correction affects tensile stiffness, and show how slack correction may overestimate, underestimate, or have no effect on tensile stiffness at low strains, depending the geometric characteristics of the original stress-strain curve over the interval containing such low strains.

MATHEMATICS BACKGROUND - CONCAVITY OF CURVE

In calculus, a differentiable function, $y = f(x)$, is said to be concave upward if its first derivative, $f'(x)$, is increasing upon an interval and concave downward if $f'(x)$ is decreasing upon an interval. In other words, if its second derivative, $f''(x)$, is positive; then the graph (curve) is concave upward; if $f''(x)$ is negative; then the curve is concave downward. Points where concavity changes are inflection points. For a straight line described by $y = ax + b$, where a and b are constants, its second derivative $f''(x) = 0$. Therefore, by definition, a straight line is neither concave upward and nor downward. Every point on a straight line is an inflection point.

THEORETICAL ANALYSIS

Figure 2 shows that a hypothetical original load-displacement curve initially concave upward up to point E then becomes concave downward. The slope, K_{OB} , of the straight line between points O and B on the original curve is defined as follows:

$$K_{OB} = y_i / x_i \tag{3}$$

The slope, K_{OC} , of the straight line between points O and C on the slack-corrected load-displacement curve is the same as K_{AD} of the straight line between points A and D on the original curve provided that $\Delta x = x_o$. K_{AD} can be expressed as follows:

$$K_{AD} = (y_i + \Delta y - y_o)/(x_i + \Delta x - x_o) \tag{4}$$

In order to have a valid comparison of K_{AD} and K_{OB}, Δx must be equal to x_o. Equation 4 can be rewritten as follows:

$$K_{AD} = y_i / x_i + (\Delta y - y_o)/ x_i \tag{5}$$

Combining Equations 3 and 5 gives:

$$K_{AD} = K_{OB} + (\Delta y/\Delta x - y_o/\Delta x)(\Delta x/ x_i) \tag{6}$$

Since Δx or x_o is small with respect to x_i, $\Delta y/\Delta x$ and $y_o/\Delta x$ can be approximately expressed as follows:

$$\Delta y/\Delta x = f'(x_i + x_o) \quad (7)$$

$$y_o/\Delta x = f'(x_o) \quad (8)$$

Combining Equations 6, 7, and 8 gives:

$$K_{AD} = K_{OB} + [(f'(x_i + x_o) - f'(x_0)] (\Delta x/ x_i) \quad (9)$$

It is noted that $\Delta x/ x_i$ is always greater than zero. For an initially concave upward curve over the interval containing x_0 and $x_i + x_o$, the following basic relationship exists:

$$f'(x_i + x_o) > f'(x_0) \quad (10)$$

Hence:

$$\boldsymbol{K_{AD} > K_{OB}} \quad (11)$$

Therefore, the corresponding tensile stiffness calculated from the slack-corrected load-displacement curve is always greater than that calculated from the original load-displacement curve. This proves that the general perception within the geosynthetic community (i.e., slack correction overestimates tensile stiffness at low strains) is true for an initially concave-upward load-displacement curve.

It is noted that the above analysis is generalized and applicable to other types of curves (i.e., linear and concave downward) though it was based on an initially concave-upward curve. If an original load-displacement curve, *f(x),* can be characterized as linear, the linear load-displacement relationship can be described by the following equation:

$$f(x) = kx + \mathrm{b} \quad (12)$$

Where: k = slope of straight line and b = intercept of the straight line on the y axis. The first derivative $f(x)$ is as follows:

$$f'(x) = k \tag{13}$$

Equation 13 is valid for x = any value, and therefore, it can be rewritten as:

$$f'(x_i + x_o) = f'(x_0) \tag{14}$$

Hence:

$$\boldsymbol{K_{AD} = K_{OB}} \tag{15}$$

This is obvious because the slope of a straight line is always the same before or after an arbitrary shift (x_o, y_o). The corresponding tensile stiffness values calculated from the original and shifted load-displacement curves (i.e., parallel straight lines) are always the same. This shows that slack correction has no effect on tensile stiffness for a geosynthetic material that exhibits a linear stress-strain relationship. The general perception within the geosynthetic community (i.e., slack correction overestimates tensile stiffness) is thus false for a geosynthetic material that exhibits a linear load-displacement relationship.

For an initially concave downward load-displacement curve over the interval containing x_0 and $x_i + x_o$, the following basic relationship exists:

$$f'(x_i + x_o) < f'(x_0) \tag{16}$$

Hence:

$$\boldsymbol{K_{AD} < K_{OB}} \tag{17}$$

Therefore, the corresponding tensile stiffness calculated from the slack-corrected load-displacement curve is always less than that calculated from original load-displacement curve. This shows that the general perception within the geosynthetic community (i.e., slack correction overestimates tensile stiffness) is false for an initially concave-downward load-displacement curve.

EXPERIMENTAL VERIFICATION

Multi-rib or wide-width tensile tests were performed on: (i) BasXgrid 11 geogrid; (ii) BasXgrid 12 geogrid; (iii) HP370 polypropylene (PP) woven geotextile; (iv) extruded PP geogrid; and (v) high strength polyester (PET) geotextile. All of the five geosynthetic materials were provided by Mirafi Construction Products. The stress-strain curves of these five geosynthetic materials based on the original and slack-corrected load-displacement curves are shown in Figures 3 through 10. Concavities of these curves and tensile stiffness values at 1%, 2%, and 5% strains were calculated from the original and slack-corrected load-displacement curves and are summarized in Table 1. The test results summarized in Table 1 clearly support the results of theoretical analysis:

- For the concave downward load-displacement curves of BasXgrid 11 and 12 geogrids in both directions, HP370 geotextile in XD, and extruded PP geogrid in MD, tensile stiffness values at 1, 2, and 5% strain calculated from the slack-corrected load-displacement curve are less than those directly calculated from the original load-displacement curve;

- For the nearly linear load-displacement relationship of HP370 geotextile in MD, tensile stiffness values at 1, 2, and 5% calculated from the original and slack-corrected load-displacement curves are approximately the same; and

- For the concave upward load-displacement curve of the high strength polyester geotextile, tensile stiffness values at 1, 2, and 5% calculated from the slack-corrected load-displacement curve are greater than those directly calculated from the original load-displacement curve.

CONCLUSIONS

Based on the results of theoretical analysis and limited experimental results, the following conclusions can be drawn:

- If an original stress-strain curve is initially concave upward over an interval, then slack correction increases tensile stiffness at any strain level within the interval. In other words, the slack correction overestimates the tensile stiffness at any strain level within the interval.

- If an original stress-strain curve is initially concave downward over a strain interval, then slack correction reduces tensile stiffness at any strain level within the interval. In other words, slack correction underestimates the tensile stiffness at any strain level within the interval.

- If an original stress-strain curve is linear over an interval, then slack correction has no effect on tensile stiffness at any strain level within the interval.

The general perception within the geosynthetic community (i.e., slack correction overestimates tensile stiffness or secant modulus at low strains) is true only if the original load-displacement curve is initially concave upward over the interval containing those low strains. This perception is false if the original load-displacement curve is initially either linear or concave downward over the interval containing those low strains.

For BasXgrid 11 and 12 geogrids in both MD and XD, HP370 geotextile in XD, and extruded PP geogrid in MD tested during this study, the test results indicate that slack correction underestimated the tensile stiffness values at 1, 2, and 5% strains. For HP370 geotextile in MD tested during this study, the test results indicate that slack correction had almost no effect on the tensile stiffness values at 1, 2, and 5% strains. For the PET geotextile in MD tested during this study, the test results indicate that slack correction overestimated the tensile stiffness values at 1, 2, and 5% strains.

Table 1. Summary of Tensile Stiffness Values for Geosynthetics A, B, and C Determined from Original and Slack-Corrected Load-Displacement Curves

Test Material	Concavity [1]	Slack Correction x_o , y_o (in, lb)	Tensile Stiffness		
			1% Strain (lbs/ft)	2% Strain (lbs/ft)	5% Strain (lbs/ft)
BasXgrid 11 Geogrid in MD	Concave downward	None	33400	28000	21300
		0.004, 20	33000	27600	21100
BasXgrid 11 Geogrid in XD	Concave downward	None	42551	29806	20148
		0.002, 29	39147	27933	19439
BasXgrid 12 Geogrid in MD	Concave downward	None	34989	27613	20332
		0.002, 18	33145	26457	19930
BasXgrid 12 Geogrid in XD	Concave downward	None	54437	40362	28981
		0.003, 35	51539	38486	28336
HP370 Geotextile in MD	Nearly linear	None	26220	31934	32948
		0.006, 30	26328	31961	33019
HP370 Geotextile in XD	Concave downward	None	60261	48129	40497
		0.002, 24	59350	47045	40140
Extruded PP Geogrid in MD	Concave downward	None	30400	25500	18300
		0.007, 20	30200	25050	18060
Polyester Geotextile in MD	Concave upward	None	94900	140850	206640
		0.01, 50	122100	162550	215040

Note (1): Concavity of each original load-displacement curve over a specific interval was identified by visual observation and shown on each figure. MD = machine direction and XD = cross-machine direction.

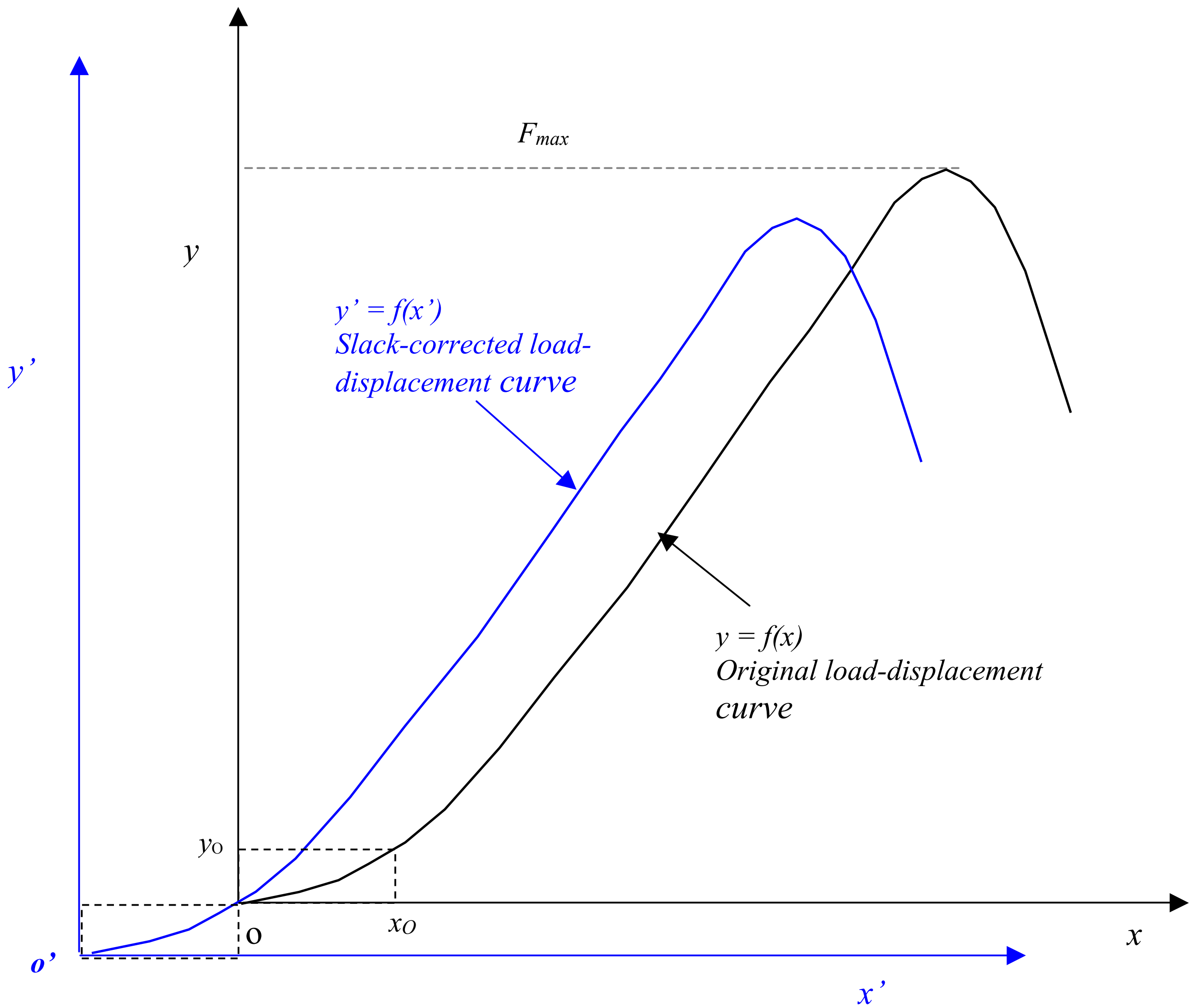

Figure 1. Slack tension and displacement defined on an original load-displacement curve

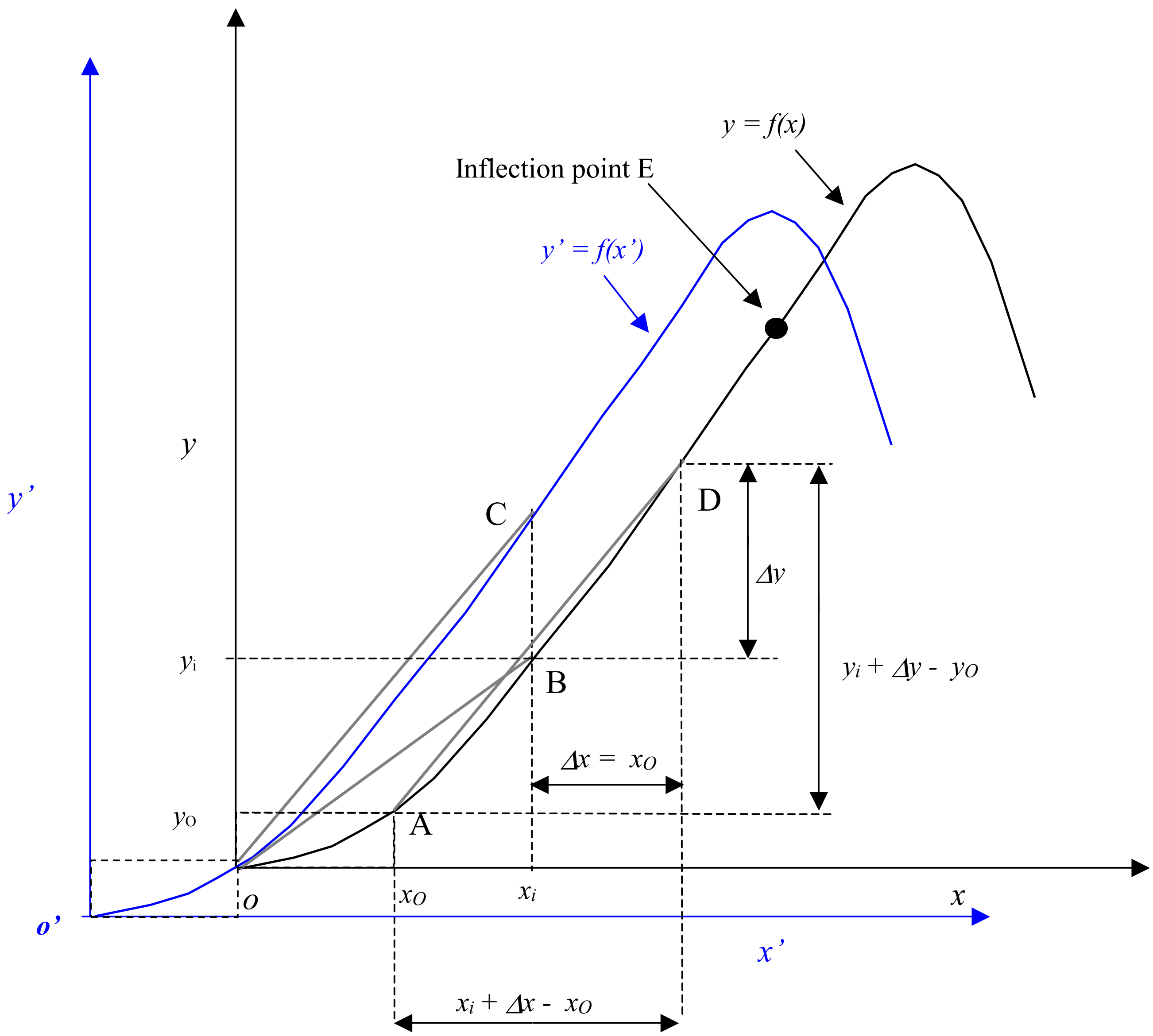

Figure 2. Slopes of an original and slack-corrected load displacement curves

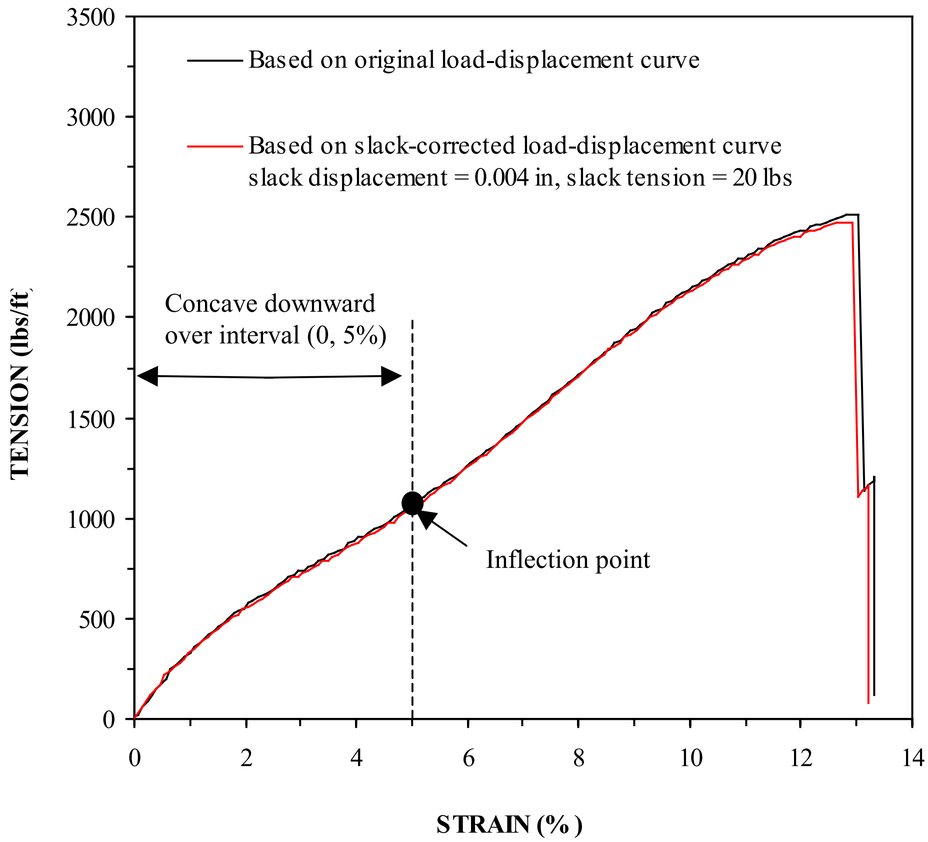

Figure 3. Stress-strain curves with and without slack correction obtained from the same test on BasXgrid 11 geogrid in the machine direction

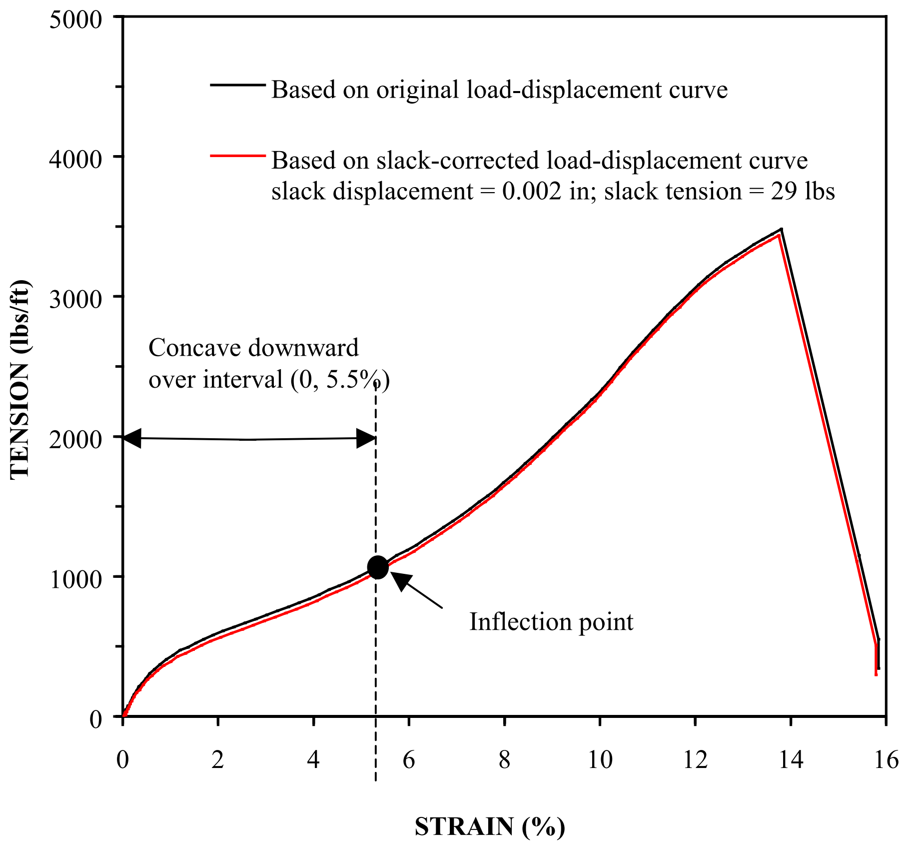

Figure 4. Stress-strain curves with and without slack correction obtained from the same test on BasXgrid 11 geogrid in the cross-machine direction

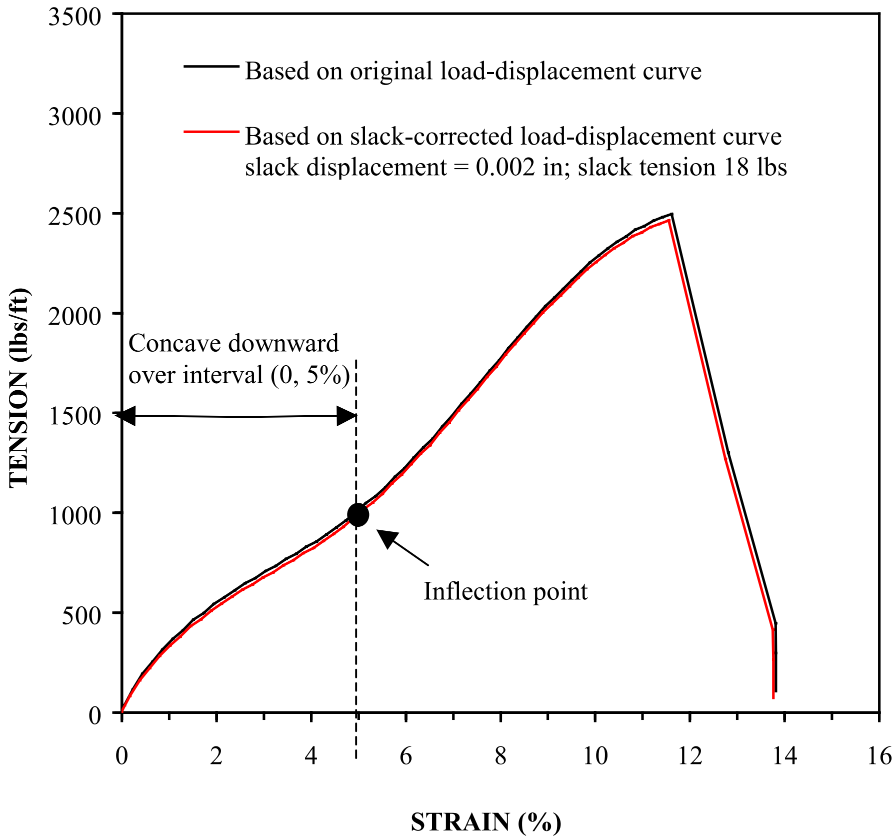

Figure 5. Stress-strain curves with and without slack correction obtained from the same test on BasXgrid 12 geogrid in the machine direction

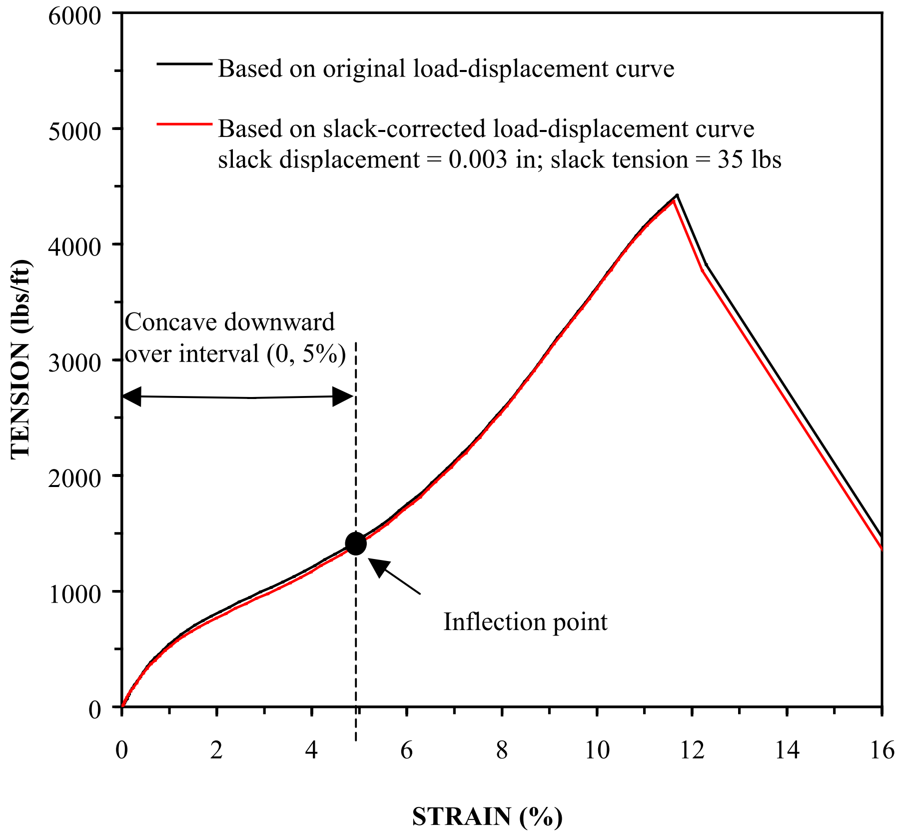

Figure 6. Stress-strain curves with and without slack correction obtained from the same test on BasXgrid 12 geogrid in the cross-machine direction

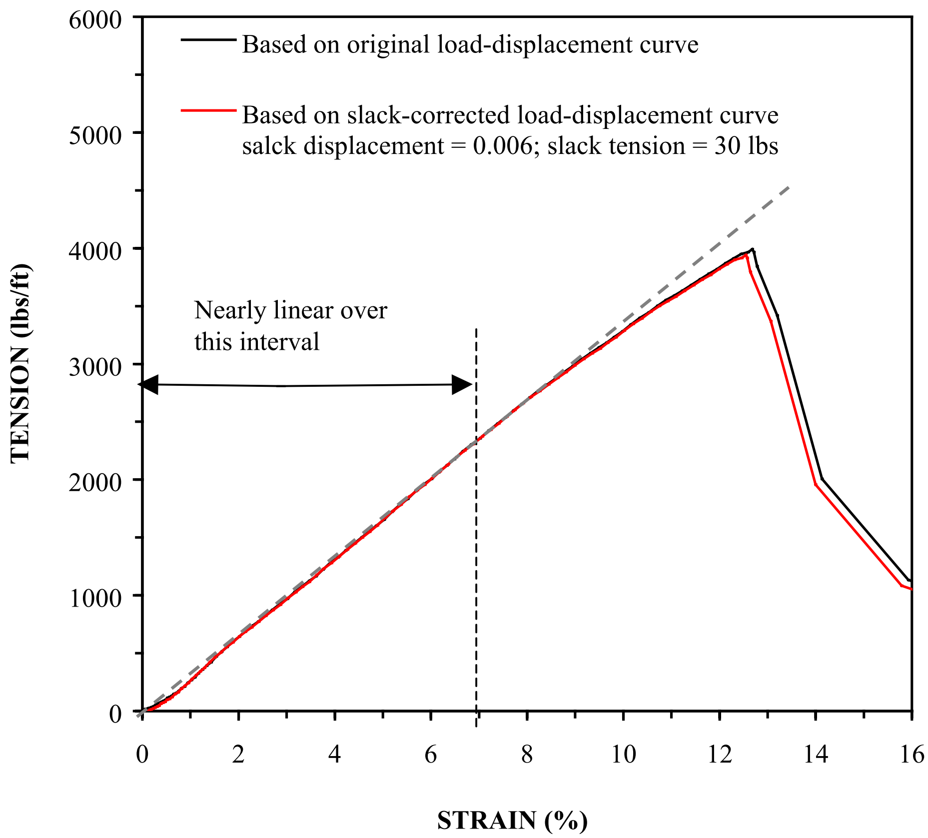

Figure 7. Stress-strain curves with and without slack correction obtained from the same test on HP370 geotextile in the machine direction

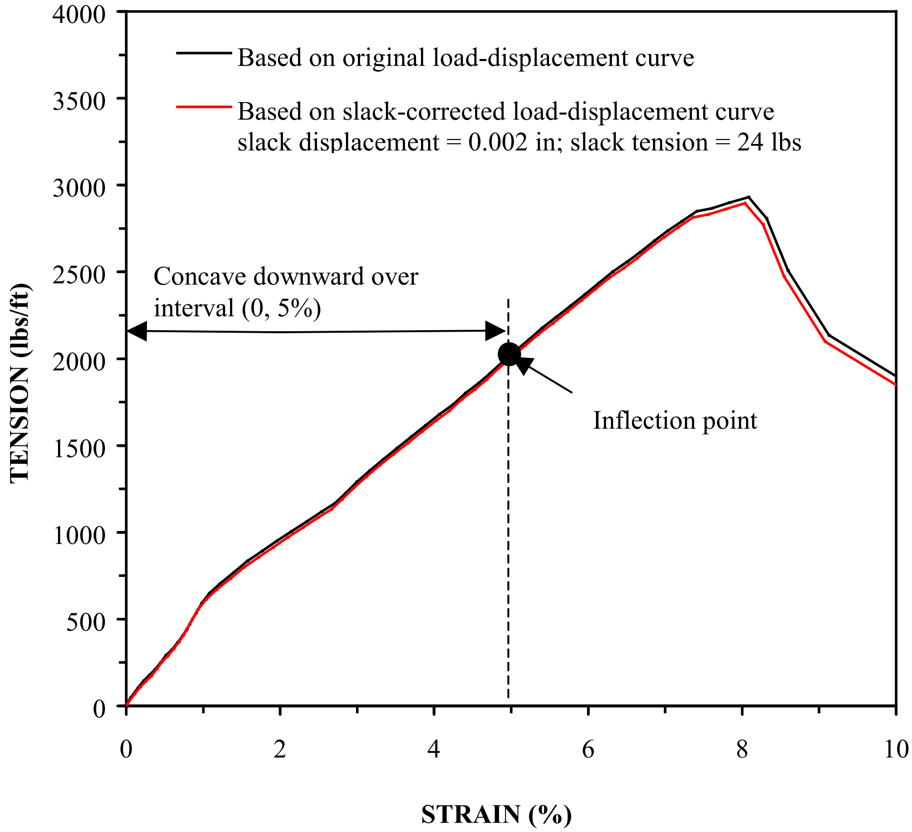

Figure 8. Stress-strain curves with and without slack correction obtained from the same test on HP370 geotextile in the cross-machine direction

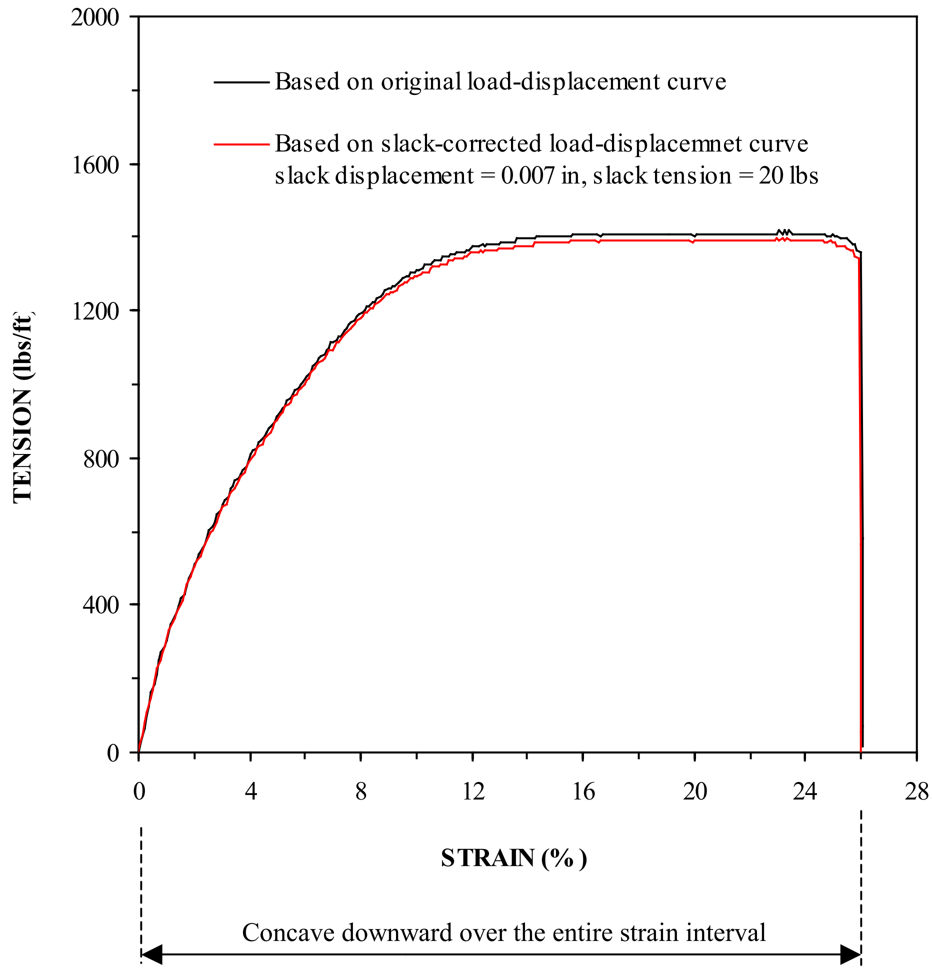

Figure 9. Stress-strain curves with and without slack correction obtained from the same test on biaxial HDPE geogrid in the machine direction

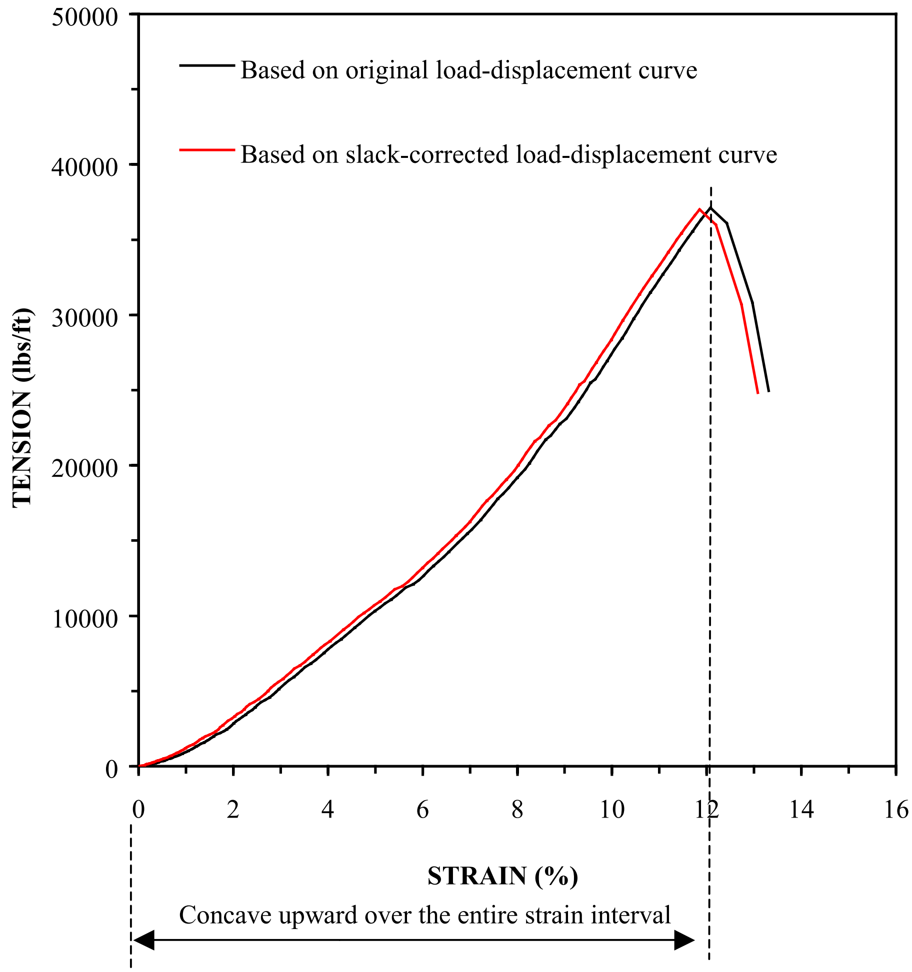

Figure 10. Stress-strain curves with and without slack correction obtained from the same test on high strength polyester geotextile in the machine direction